

## **An Integrative Wave model for the Marginal Ice Zone based on a Rheological Parameterization**

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### **LONG-TERM GOALS**

To enhance wave forecasting models such as WAVEWATCH III (WW3) so that they can predict the marginal ice zone (MIZ) wave climate in the present and future Arctic seas.

### **OBJECTIVES**

1. To build a comprehensive wave-ice interaction mathematical framework for a wide range of ice conditions observable in MIZ;
2. To identify a minimum set of rheological parameters that can reproduce all existing wave-ice interaction types;
3. To test the sensitivity of various parameters in the new wave-ice interaction model;
4. To relate physically detectable ice cover parameters from remote sensing to its rheological properties;
5. To establish a strategy for WW3 to implement the wave-ice interaction mechanisms;
6. To test the model performance and validate it using WW3.

### **APPROACH**

**For objective 1:** Complete the viscoelastic theory. Key individuals are the PI and a PhD student.

*Task 1:* Use an analytical method to determine the propagation of waves through a floating viscoelastic mat for a wide range of effective viscosity and elasticity parameters. This range should include all possible ice cover types in the MIZ. The outcome of this task is a direct relation between viscoelastic properties and the wave dispersion, including the group velocity and the attenuation.

*Task 2:* Obtain the energy flux between different regions of ice covers via analytical solutions, i.e. the transmission and reflection energy between different viscoelastic mats.

**For objective 2:** Justify the rheological parameters. Key individuals are the PI and a PhD student, in collaboration with Vernon Squire and Ben Holt.

*Task 3:* Assemble all existing laboratory and field data of wave propagation in ice covers.

*Task 4:* Determine if all existing evidence of wave property changes across ice covers is reflected in the theoretical model, i.e. verify that the proposed viscoelastic theory is capable of producing qualitatively similar range of the observations.

**For objective 3:** Test the sensitivity of parameters. Key individuals are the PI and a PhD student.

*Task 5:* Apply statistical analysis to examine the sensitivity of model dependence on the rheological parameters. It is likely that over certain ranges of parameters the resulting wave property change may either be highly sensitive or insensitive to changes of the parameter values. This part of the study will help to simplify the wave-ice interaction model.

**For objective 4:** Relate accessible ice cover data to its rheological properties. Key individuals are the PI, a postdoc, and a graduate student, in collaboration with Vernon Squire. Additional information will be collected from all other PIs in this DRI.

*Task 6:* Perform inverse analysis to map accessible wave dispersion and reflection/transmission data to the viscoelastic parameters. The accessible data will include all old and new data from field, laboratory, and remote sensing studies, as well as those derived from the scattering theory.

**For objective 5:** Develop tools to implement the rheological model into WW3. Key individuals are the PI, a postdoc, and a PhD student.

*Task 7:* Determine the best strategy to implement the rheological theory to WW3. The mathematical theory contains many parameters. Each set of parameters produces different wave group velocity, attenuation coefficient, and transmission and reflection properties. Results from the theory are input that needs to feed into WW3. These results are both time and space dependent. Implementation method will be established keeping in mind the resolution, accuracy, and computational efficiency.

**For objective 6:** Test the updated WW3. Key individuals are the PI, a postdoc, and Erick Rogers.

*Task 8:* The modified WW3 will be tested using both hind- and fore-casts.

## **WORK COMPLETED**

Tasks 1, 2, 3 have been completed. Tasks 4, 5, 6 continue to make progress. Task 7 has been further improved. Task 8 is currently being carried out by Erick Rogers. We have also tried it using two sets of field data obtained in 2000 and 2012 in the Antarctic to determine its performance under calm and storm conditions.

*Tasks 1,2:* The viscoelastic theory gives both the change of wave speed and wave attenuation once the equivalent mechanical properties of an ice cover are determined. This theory yields several possible wave modes (Wang and Shen, 2010). The interesting mathematical structure and its physical implications are under further investigation. Until proven necessary, the leading mode of the dispersion relation is solved and implemented in WW3 as option IC3 described in the manual (Tolman et al. 2014). Instead of using a look-up table as first envisioned, a fast numerical procedure is used to directly solve the dispersion relation within WW3. The code provided last year however crashed under long wave conditions. This problem has been identified as resulting from the initial guess of the process of solution procedure. It has been rectified in the new code recently submitted to Erick Rogers. The transmission/reflection between two different viscoelastic regions is further studied to include more modes and to improve the matching boundary conditions between two different regions. The

results show, fortunately, that the approximate method using two leading modes is not significantly different from the more rigorous approach. This transmission/reflection mechanism has not been implemented in WW3.

*Tasks 3:* Two field data sets both in the Antarctic marginal ice zone were used to evaluate the viscoelastic ice damping models. The 2012 data came from two buoys separated by over 100km most of the time (Kohout et al. 2014). The WW3 hindcast was successfully performed to compare with this data set. The 2000 data set however was from buoys separated by at most 25km (Doble et al. 2003, Doble and Wadhams, 2006). This distance is about the resolution of the ice and wind data, hence unless finer resolution ice and wind data can be obtained, it is difficult to use the 2000 data to separate the sea ice attenuation from other source terms. One recent laboratory data was used to determine the viscoelastic parameters. These results have been published (Zhao and Shen, 2015b).

*Tasks 4,5,6:* Due to the scarcity of the datasets, using the available information to inversely determine the ice cover properties presents large uncertainty. Sensitivity analysis based on ANOVA is completed and a paper reporting this analysis has been accepted for publication (Li et al. to appear).

*Task 7:* This task is now further improved so that the speed of running the viscoelastic model is only 16% more than the IC1 module which assumes a constant damping for all frequencies.

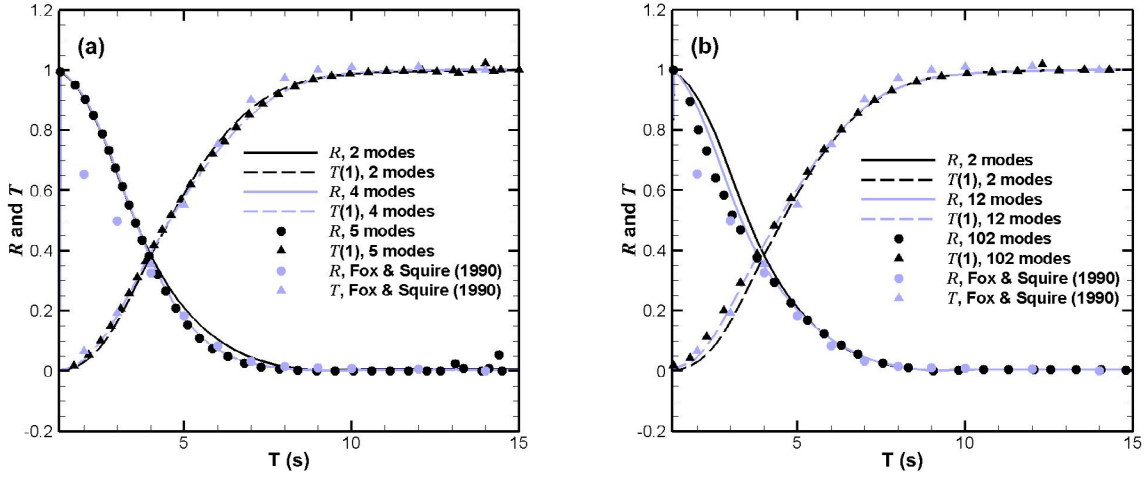
*Task 8:* Erick Rogers has begun comparing results from three different rheological models for the ice cover: constant attenuation, eddy-viscosity, and viscoelastic. We have begun learning using WW3 so to conduct parameter optimization. We have found that other source terms such as the wind input, nonlinear wave interaction, and damping may influence the interpretation of the ice attenuation. In some cases, the full WW3 simulation is needed to determine the best fit viscoelastic parameters from the field data. We have published a paper based on the 2012 data from the Southern Ocean on these findings (Li et al. 2015).

The research team is now preparing for the field campaign to take place Oct. 1-Nov. 10 this year. We will have abundant data afterwards to digest. This project is truly a synthesis of data and models from a large number of sources. We have tried to conduct such a complex study by using the 2014 field data to study wind/wave effects on the ice morphology during the summer-fall transition, in collaboration with Ben Holt, Erick Rogers, and Jim Thomson. A paper is being prepared for submission.

## RESULTS

### 1) Theoretical development

A more rigorous solution matching ice covers of different properties was developed to solve the wave propagation between different types of ice cover (Zhao and Shen, 2015c). This work is published. An example is given in Fig. 1, which shows the effect of including multiple roots in the solution. We have found that including multiple roots does not change the results much, hence for applications at most two roots should be sufficient.



**Fig. 1. Effect of including additional modes on transmission/reflection from open water to an elastic sheet with the same parameters as in Fig. 2. (a) 2 modes: include 2 propagating modes; 4 modes: add 2 symmetrical damped propagating modes; 5 modes: add 1 additional propagating mode. (b) 12 modes: add 10 evanescent modes to the 2 modes case. 102 modes: add 100 evanescent modes to the 2 modes case. Fox and Squire's results used 100 evanescent modes. (From Zhao and Shen, 2015c.)**

## 2) Sensitivity analysis

An inverse method is planned after the field trip to determine the best fit viscoelastic parameters between the model and the measured data. For this method to be successful, we need to evaluate the sensitivity of the model to its parameters. We completed such a study using ANOVA, a standard statistical package to evaluate a multi-variable system, where parameter interactions may affect the level of sensitivity. Table 1 provides a key result, where the sensitivity of wave number and attenuation rate in response to the ice parameters, depth of the sea, and wave period are given. The higher the  $F$  value, the more sensitive the parameter itself, or the interaction of the pair parameters shown is. Using an inverse method in these regions needs to be more careful. The same study is also conducted for the viscoelastic model proposed by Squire and Allen (1980). It was found that both viscoelastic model behave the same way, although the effective elastic and viscous parameters from the two models can be orders of magnitude different (Li et al. to appear).

**Table 1. The analysis of variance table for the viscoelastic model. High  $p$  values mean low sensitivity. High  $F$  value means high sensitivity. (From Li et al., to appear.)**

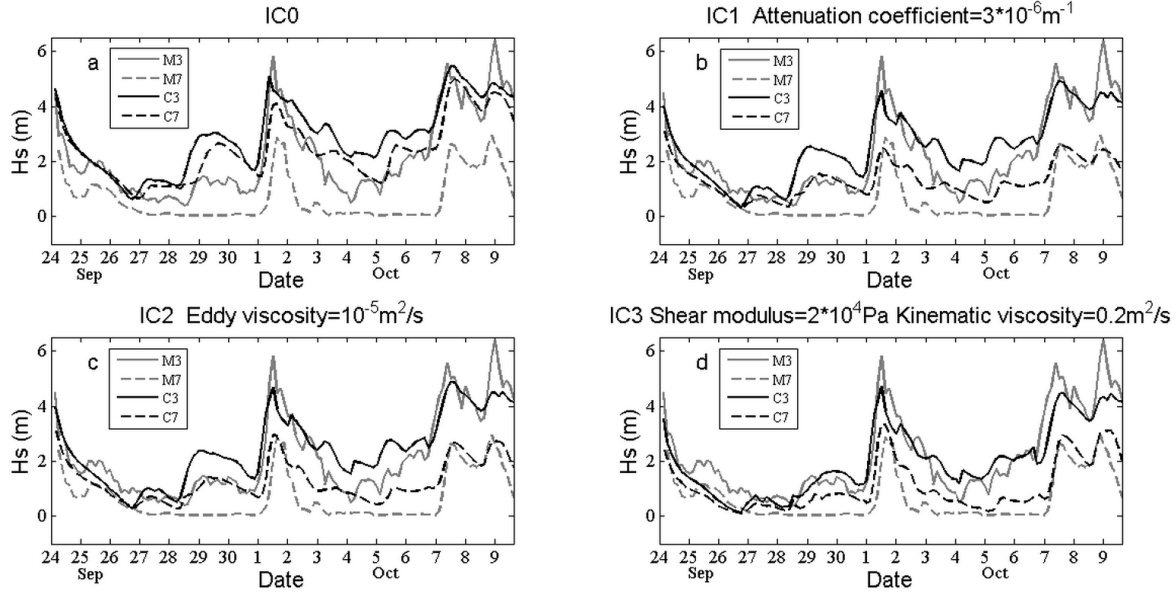
Response: Normalized wave number $\kappa$			Response: Attenuation coefficient $\mathcal{Q}$		
Source	$F$	$p$	Source	$F$	$p$
$T$	1498.60	0.000	$T$	1364.29	0.000
$H$	7.00	0.000	$H$	0.85	0.492
$h$	107.33	0.000	$h$	162.44	0.000
$G$	3641.88	0.000	$G$	292.48	0.000
$v$	0.63	0.643	$v$	108.64	0.000
$T * H$	0.76	0.731	$T * H$	1.50	0.090
$T * h$	186.33	0.000	$T * h$	197.59	0.000
$T * G$	1983.73	0.000	$T * G$	644.96	0.000
$T * v$	0.99	0.469	$T * v$	150.48	0.000
$H * h$	2.13	0.012	$H * h$	1.55	0.098
$H * G$	1.88	0.018	$H * G$	1.50	0.090
$H * v$	0.55	0.924	$H * v$	1.31	0.182
$h * G$	371.02	0.000	$h * G$	35.08	0.000
$h * v$	0.60	0.848	$h * v$	10.68	0.000
$G * v$	0.51	0.946	$G * v$	38.28	0.000

### 3) Field data

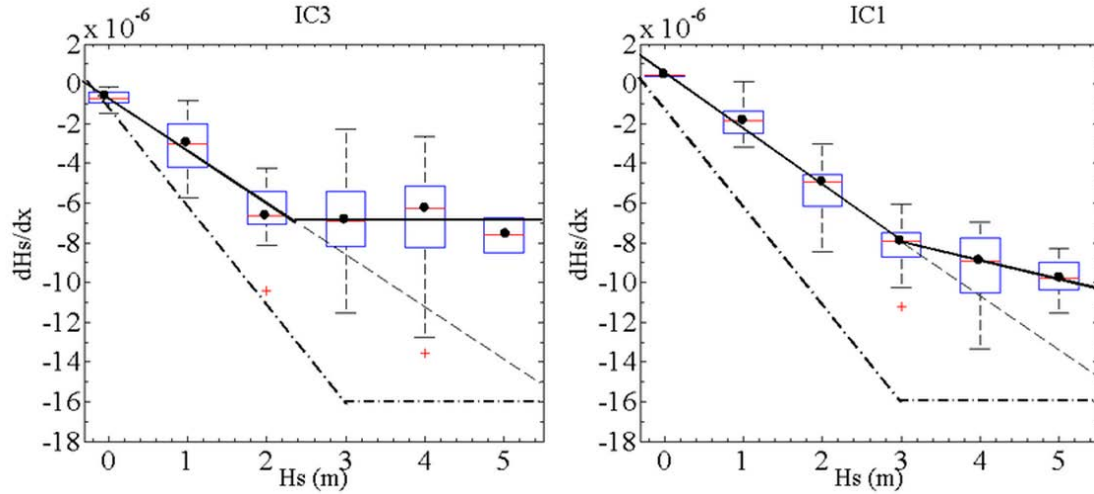
A data set containing both calm and storm cases of wave conditions from two drift buoys was used to determining the contributions from the four major wave source/sink terms on the right hand side of the wave action equation below: wind input, nonlinear wave interaction, turbulence/wave breaking, and sea ice attenuation, respectively (Tolman et al. 2014).

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial x}(C_{g,x}N) + \frac{\partial}{\partial y}(C_{g,y}N) + \frac{\partial}{\partial k}(C_k N) + \frac{\partial}{\partial \theta}(C_\theta N) = S_{in} + S_{nl} + S_{ds} + S_{ice} \quad (1)$$

The modeled significant wave height from four different sea ice attenuation modules, IC0, IC1, IC2, and IC3 is compared to the field data from Kohout et al. (2014) in Fig. 2. It appears that IC3 is able to have the closest overall agreement. Also, the apparent attenuation rate as shown in Fig. 3 shows that between calm and storm cases, there is a change of the apparent attenuation coefficient without any change of the sea ice attenuation formulation. This is because all other three source terms have very different levels of effect between calm and storm conditions. Particularly the wind input and the nonlinear wave interaction become orders of magnitude more important in storm cases. Thus they mask the sea ice damping, and changed the apparent attenuation rate. This conclusion has been speculated in Wadhams et al. (1988), and theoretically proven in Masson and LeBlond (1989). It is now also verified by a combination of field data and WW3 model results (Li et al. 2015).



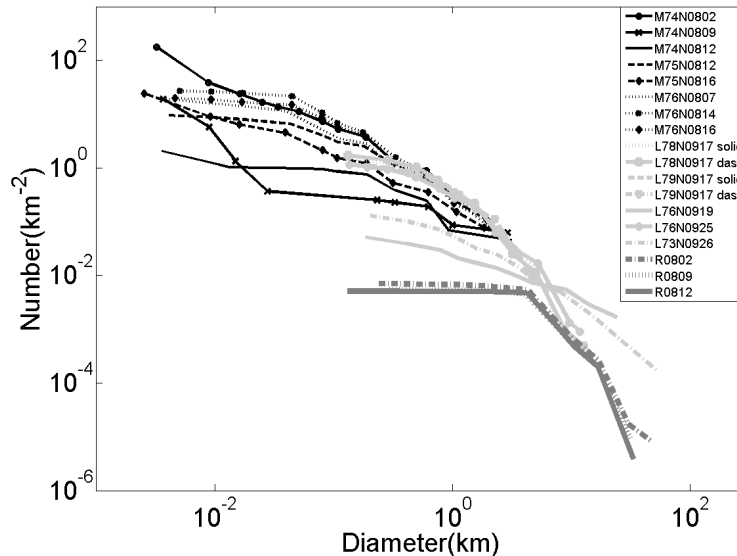
**Fig. 2. Comparisons of measured and simulated significant wave height  $H_s$  of the sensor closest to the ice edge (sensor 3, measured: solid gray, simulated: solid black) and the sensor farthest from the ice edge (sensor 7, measured: dash gray, simulated: dash black) with different ice damping methods. (From Li et al. 2015.)**



**Fig. 3. Decay rates of sensors farther than 100 km from the ice edge calculated by WW3 with IC3 and IC1. As was done in Figure 2 of Kohout et al (2014), the black dot is the median, box height shows the range within which 50% of the data lie. The whiskers give the range of data, excluding outliers (crosses). The solid line is calculated from linear least-squares regression through the median values. The dashed line shows the decay that would be expected if small-amplitude wave theory held for large waves. The dash-dot line is the median value of the observed  $dH_s/dx$ . (From Li et al. 2015.)**

#### 4) Remote sensing ice floe size

Working with Ben Holt, a visiting graduate student has been exploring using the MEDEA, Landsat8, and RADARSAT-2 images obtained in Aug.-Sept., 2014 with resolutions of 1m, 15m, and 100m to study the floe size distribution and leads characteristics. These data are examined against the wave buoy and model data from Jim Tomson and Erick Rogers, and wind, temperature data from various sources. The results are organized into a manuscript which is at its final preparation stage for submission. This synthesis of data types and sources is a useful preparation for the incoming field campaign. A preliminary result of the cumulative floe size distribution is shown in Fig. 4.



**Fig. 4. Cumulative floe size distribution from MEDEA, Landsat8, and RADARSAT-2.**

## IMPACT/APPLICATIONS

More accurate wave models are necessary tools for many naval operations and environmental protection purposes: such as navigation route planning, offshore structure design in the Arctic, and coastal erosion prevention. They may also be coupled with:

1. the ocean circulation models to study the effects of a more dynamic upper surface on the water body underneath;
2. the ice models to study the evolution of floe size distribution from the wave fracturing process;
3. the thermodynamic models to evaluate the melting rate from mechanical floe size reduction;
4. the coastal erosion models of the vulnerable permafrost Arctic coastal zones.

## TRANSITIONS

An improved FORTRAM subroutine for IC3 module has been generated and delivered to the WW3 group.

## RELATED PROJECTS

A related project funded by the Singapore Ministry of Education Academic Research Fund (AcRF) Tier 2 began in January 2014. The project, entitled: “Wave drift and attenuation of viscoelastic floating substances”, is led by the principal investigator Prof. Adrian Wing-Keung Law ([http://research.ntu.edu.sg/expertise/academicprofile/pages/StaffProfile.aspx?ST\\_EMAILID=CWKLA W&CategoryDescription=watersustainability](http://research.ntu.edu.sg/expertise/academicprofile/pages/StaffProfile.aspx?ST_EMAILID=CWKLA W&CategoryDescription=watersustainability)). The PI, Hayley Shen, is the international collaborator in the project which will run from Jan. 2014-Dec. 2016. A small wave flume (30cm wide and 8m long) is completed at the end of 2014. Equipment for making PDMS (a viscoelastic material) with adjustable and precisely measured viscoelastic properties has been in operation and successfully used to produce a wide range of samples. The stability of these samples has been established in extensive laboratory studies performed by a postdoc. Several wave tests have been conducted by a PhD student. This project will serve as a necessary check for our wave-ice interaction project. The data obtained will be useful to first validate the theory and then used as a true “continuum” analog of the fragmented ice covers over large scale. In addition, a proposal has been submitted by Prof. Law to ONR-Global. This proposal entitled “Wave impact on Arctic Shipping and Offshore Technology – A unique modeling facility” will use the above mentioned facility to further validate the viscoelastic theory and explore more complex systems of layered materials that could simulate natural systems in the Arctic environment.

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## **PUBLICATIONS**

### **Journal articles –**

1. Zhao, X., Shen, H.H., and Cheng, S. (2015a) Modeling ocean wave propagation under sea ice covers Review Paper ACTA MECHANICA SINICA DOI:10.1007/s10409-015-0017-5
2. Zhao, X and Shen, HH (2015b) Wave propagation in frazil/pancake, pancake, and fragmented ice covers, Cold Regions Science and Technology, doi:10.1016/j.coldregions.2015.02.007
3. Zhao, X and Shen HH (2015c) Ocean wave transmission and reflection by viscoelastic ice covers, Ocean Modelling, doi:10.1016/j.ocemod.2015.05.003.
4. Li, J., A.L. Kohout, and H.H. Shen (2015) Comparison of wave propagation through ice covers in calm and storm conditions. Geophy. Res. Letts. 42(14): 5935–5941, doi: 10.1002/2015GL064715.

### **Conference abstracts –**

5. Shen, H.H., Zhao, X., Callinan, C., Wilkinson, J., and Evers, K.-U. (2014) Laboratory Experiment of Wave Propagation Through Different Ice Covers, AGU Fall Meeting, San Francisco, Dec. 15-19, 2014, Poster #C11A-0351.
6. Zhao, X., and Shen, H.H. (2015) Wave scattering by randomly distributed ice floes: diffusion approximation, 2015 AGU Joint Assembly Meeting, May 3–7, 2015, Montreal, Canada. Abstract #OS33A-02.
7. Li, J. Mondal, S., and Shen, H.H. (2015) Sensitivity analysis of the viscoelastic wave-in-ice model, 2015 AGU Joint Assembly Meeting, May 3–7, 2015, Montreal, Canada. Abstract #OS33A-03.
8. Cheng, S., and Shen, H.H. (2015) Characterizing the behavior of gravity wave propagation into a floating or submerged viscous layer, 2015 AGU Joint Assembly Meeting, May 3–7, 2015, Montreal, Canada. Abstract #OS33A-04.
9. Wang, Y., Holt, B., Rogers, E.W., Thomson, J., and Shen, H.H. (2015) Characteristics and changes of sea-ice floe size distribution in Chukchi and Beaufort Seas in fall 2014, 2015 AGU Joint Assembly Meeting, May 3–7, 2015, Montreal, Canada. Abstract #OS33A-05.